Technical University Cottbus
Institute of Mathematics

Dr. rer. nat. habil. Ursula Felgenhauer Academic Year 2004/05

Tutorial **Numerical Treatment of Optimal Control Problems**

Motivation. Optimal control problems occur in modern applied sciences in rather different fields, e.g. techno-mechanical systems, in economy and ecological studies, in medicine and life-sciences. From their mathematical nature, optimal control can be characterized as a branch of optimization in function spaces, or infinite-dimensional optimization. The numerical treatment, as a rule, requires in essence two approximation steps: first, the construction of finite-dimensional subproblems via discretization, and secondly, the iterative solution of the nonlinear systems by Newton-type or related methods.

– In the first talk, we consider typical optimal control problem formulations. Beginning with an classical example, the time-optimal termination of a harmonic oscillator, some basic definitions are introduced. By the Lagrangian formalism, the adjoint equations are obtained and discussed together with the maximum condition.

– The second talk is devoted to Pontryagin's maximum Principle. For a certain autonomous Mayer type problem, the idea of needle variations is explained, and the proof is sketched together with some sensitivity information. Several notations of the resulting stationarity system are given. The analysis is completed by a discrete formulation of the maximum principle.

– The topic of the third lecture is the Lagrange-Newton approach. First, we give an abstract introduction for optimization problems in a Banach space situation. In a second step, a SQP iteration is given for control-constrained optimal control problems. An example shows, how the problem can be treated with standard optimization software (e.g. MATLAB).

– In Lecture 4, the shooting method for control problems is considered. Starting from the simple one-stage shooting idea, we show how multiple shooting procedures are constructed. The connection to the Newton approach is enlightened, and applications to different constraint types are shortly explained.

In the conclusion, modern developments in the field are mentioned.

Cottbus, in April 2005.