Parameter Estimation in Nonlinear Models

Series of four lectures by
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Abstract

Estimating parameters in nonlinear model equations from observations of the variables which are subject to errors is a basic task when applying mathematics to real life problems. Typically, the model equations are algebraic or differential equations. They describe the relations between the variables which are the state variables of the underlying process and/or the time and/or space coordinates and contain unknown parameters to be estimated.

In a series of four talks a survey of both the different types of models as well as numerical methods for solving the resulting optimization problems will be given.

In the first talk, the explicit model equation $y = r(x, p)$ is considered including the practically relevant special cases that the response function r is linear in both x and p (*linear models*), linear in p (*parameter linear models*), or linear with respect to some parameters (*semi-linear models*, also called *models with separated variables*). In the *regression case*, the independent variables x are supposed to be observed without errors. If they are subject to errors, too, than one has *models with errors in the variables* or *orthogonal regression problems.* In both cases the least squares principle leads to unconstrained structured nonlinear least squares problems.

In the second talk, basic numerical methods for solving linear least squares problems will be discussed. At first, solvability, sensitivity, and ill-posedness will be investigated based on the Singular Value Decomposition (SVD). Then numerical methods as the normal equation approach and orthogonalization methods are shortly sketched. Finally, discrete and continuous regularization techniques for ill-conditioned and rank-deficient problems will be discussed.

In the third talk, Gauss-Newton and Newton approaches for solving nonlinear least squares problems are introduced. Their local convergence properties are analyzed, and several globalization techniques like *descent by line search* or *trust region techniques* and the alternative *homotopy techniques* are discussed.

In the fourth talk, orthogonal regression problems are considered. For linear models one ends up with *total least squares problems* which can be solved using a SVD. For nonlinear models, the corresponding problems have large dimension but special structure which allows using special *decomposition techniques.* Finally, as outlook, parameter estimation in *implicit model equations* $f(x, y, p) = 0$ which leads to equality constrained least squares problems will shortly be addressed.