M-matrices and discretization methods

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Abstract

There is an intimate relationship between nonsinguler M-matrices and the discretization of second order linear elliptic and parabolic problems by finite difference methods, finite element methods and the method of lines.

In the first part of the lectures, the class of M-matrices is introduced and important M-matrix properties are discussed with an eye towards subsequent application in discretization methods.

With the focus on maximum principles and inverse monotonicity, a one-dimensional problem is usually introductury. For it, we construct exact three-point discretizations by different methods and analyse the resulting linear equation systems which involve tridiagonal M-matrices. From this point of view, we turn also to a number of standard approximations by FDM and FEM for the same bondary value problem.

As a second example, we briefly check some approximations of a two-dimensional elliptic boundary value problem, where basic qualitative properties of the resulting linear equation systems are again of main interest.