Fakultät Mathematik und Naturwissenschaften

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Syllabus for the course

Optimal Control with PDEs

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Optimal Control with Partial Differential Equations

The aim of the course is to provide a numerically oriented approach to variational inequalities in Hilbert spaces and there discretization by finite element methods. The analysis as well as the numerical methods are primarily oriented towards optimal control problems with partial differential equations in its weak formulation, i.e. with state equations as variational equations.

The 60 h course aimed for graduate students and PhD-students. As prerequists are required: Real analysis, basics of functional analysis and numerical analysis. The course contains about

32 h lectures, 20 h tutorials, 8 h computer lab.

In detail the syllabus is:

1 Variational equations and inequalities in Hilbert spaces (6 h + 4 h)

- 1.1 Examples of important partial differential equations and of differential inequalities
- 1.2 Weak derivatives and Sobolev spaces H^1, H_0^1 and H^2, definition, properties, embedding theorems
- 1.3 Weak formulations for differential equations
- 1.4 Generation of variational inequalities as optimality criterions
- 1.5 Existence and uniqueness of solutions, Lax-Milgram Lemma

2 Finite element discretization of variational equations (6 h + 4 h)

- 2.1 Conforming discretization, existence and uniqueness of discrete solutions
- 2.2 Convergence of discrete solutions, Cea's Lemma
- 2.3 Construction of triangular elements in $\mathbb{R}^2$
- 2.4 Error estimates via the Lemma of Bramble-Hilbert

3 Optimal control problems with PDEs (6 h + 4 h)

- 3.1 Control problems of tracking type
- 3.2 Solution operator of the state equation and its adjoint
- 3.3 Bounds on controls, existence of optimal multiplier functions
- 3.4 Restrictions upon states, regularity properties

4 Discretization of optimal control problems with PDEs (6 h + 4 h)

4.1 Discretization of the solution operator of the state equation and its adjoint
4.2 Discrete treatment of bounds on controls
4.3 Full discretization versus control reduction
4.4 Discretization of state constraints, regularity properties

5 Penalty methods to treat control and state restrictions (4 h + 2 h)

5.1 Families of penalties and its convergence properties
5.2 Smoothed exact penalties for discrete control bounds
5.3 Outer approximation by quadratic penalties
5.4 Approximate control reduction
5.5 Mesh-indedendence properties of penalty methods

6 Iteration-projection methods (4 h + 2 h)

6.1 Gradient type methods with projections
6.2 Families of nested finite element spaces
6.3 Inner approximations of fixed point sets

7 Numerical experiments using MATLAB (8 h)

Literature

[1] Tröltzsch, F.: Optimal control of partial differential equations. Theory, methods and applications. Graduate Studies in Mathematics 112. Providence, RI: AMS, 2010.

[2] Grossmann, C.; Roos, H.-G.; Stynes, M.: Numerical treatment of partial differential equations. Berlin: Springer, 2007.

[3] Glowinski, R.; Lions, J.-L.; Tremolieres, R.: Numerical analysis of variational inequalities. Amsterdam, New York, Oxford: North-Holland Publ. Co., 1981.

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